

# Effects of Chemical Potential on Hadron Masses in the Phase Transition Region \*

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We study the response of hadron masses with respect to chemical potential at  $\mu = 0$ . Our preliminary results of the pion channel show that  $\partial m / \partial \mu$  in the confinement phase is significantly larger than that in the deconfinement phase, which is consistent with the chiral restoration.

## 1. Introduction

As suggested by QCD sum rule analysis [1], hadron masses may be affected by density effects. This may explain some results of heavy ion collision experiments such as dilepton spectra and  $J/\Psi$  suppression.

It is difficult to introduce density effects in lattice QCD calculations due to the well-known “complex action” problem. Here we calculate the response of hadron masses to chemical potential,  $\partial m / \partial \mu$ , on dynamical configurations with  $\mu = 0$ . Since simulations are done at  $\mu = 0$ , there is no difficulty in obtaining  $\partial m / \partial \mu$ . We investigate the dependence of  $\partial m / \partial \mu$  with the temperature.

## 2. Formulation

We use 2 flavors of staggered quarks. The effective action to simulate  $N_f$  fermion flavors is

$$S_{eff} = S_G + S_F \quad (1)$$

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where  $S_G$  is the standard plaquette action and

$$S_F = \frac{N_f}{4} \text{Tr} \ln M(U, \mu) \quad (2)$$

where  $M(U, \mu)$  is the staggered fermion Matrix.

The zero momentum hadron correlation function  $G(t)$  is given by

$$G(t) = \sum_x \langle H(x, t) H(0, 0)^\dagger \rangle \quad (3)$$

and

$$\begin{aligned} & \langle H(x, t) H(0, 0)^\dagger \rangle \\ &= \int dU H(x, t) H(0, 0)^\dagger \exp(-S_{eff}) / Z \end{aligned} \quad (4)$$

where  $Z$  is the partition function.

Taking a derivative of the hadronic correlator with respect to  $\mu$ .

$$\begin{aligned} \frac{\partial \langle H(x, t) H(0, 0)^\dagger \rangle}{\partial \mu} &= \langle \frac{\partial C(x, t)}{\partial \mu} \rangle \quad (5) \\ &= \langle C(x, t) \frac{\partial S_F}{\partial \mu} \rangle + \langle C(x, t) \rangle \langle \frac{\partial S_F}{\partial \mu} \rangle. \end{aligned}$$

where  $C(x, t) = H(x, t)H(0, 0)^\dagger$ . We calculate eq. (5) on dynamical configurations with  $\mu = 0$ . In the case of  $\mu = 0$  eq. (5) can be simplified using the following facts:

(A)  $\partial S_F / \partial \mu$  corresponds to the fermion number operator. Thus, the average of the fermion number operator at  $\mu = 0$  is zero:  $\langle \frac{\partial S_F}{\partial \mu} \rangle = 0$ .

(B) On each configuration the value of  $\partial S_F / \partial \mu$  is purely imaginary [2]. Thus, the value of  $\langle C(x, t) \frac{\partial S_F}{\partial \mu} \rangle$  is also purely imaginary provided that the operator  $C(x, t)$  is real. This is indeed the case if we consider  $C(x, t)$  for mesons made up of degenerate quarks.

Using the facts (A) and (B) above we derive

$$\frac{\partial \langle H(x, t)H(0, 0)^\dagger \rangle}{\partial \mu} = \langle \frac{\partial C(x, t)}{\partial \mu} \rangle \quad (6)$$

for mesons consisting of degenerate quarks.

In the spectral representation,

$$G(t) = \sum_i A_i \cosh(m_i(t - N_t/2)). \quad (7)$$

Taking a derivative of eq. (7) with respect to  $\mu$  we obtain

$$\begin{aligned} \frac{\partial G(t)}{\partial \mu} &= \sum_i \left[ \frac{\partial A_i}{\partial \mu} \cosh(m_i(t - N_t/2)) \right. \\ &\quad \left. + \frac{\partial m_i}{\partial \mu} A_i (t - N_t/2) \sinh(m_i(t - N_t/2)) \right]. \end{aligned} \quad (8)$$

Our procedure to obtain  $\partial m / \partial \mu$  is as follows. First we determine  $A_i$  and  $m_i$  by fitting correlation function data to eq. (7). Substituting the values of  $A_i$  and  $m_i$  into eq. (8) we fit the data of  $\frac{\partial G(t)}{\partial \mu}$  to eq. (8). Then we obtain  $\partial m_i / \partial \mu$  and  $\partial A_i / \partial \mu$  as fitting parameters.

### 3. Definition of $\partial / \partial \mu$

We study the two flavor case ( $u$  and  $d$  quarks). In this case, we have two independent chemical potentials,  $\mu_u$  and  $\mu_d$ . Instead, the following combinations are convenient,  $\mu_S = (\mu_u + \mu_d)/2$  and  $\mu_V = (\mu_u - \mu_d)/2$  which  $\mu_S$  the usual chemical potential corresponding to baryon number. Then derivatives with respect to  $\mu_S$  and  $\mu_V$  are

$$\frac{\partial}{\partial \mu_S} = \frac{\partial}{\partial \mu_u} + \frac{\partial}{\partial \mu_d} = \frac{\partial}{\partial \mu_u} - \frac{\partial}{\partial \mu_{\bar{d}}} \quad (9)$$

$$\frac{\partial}{\partial \mu_V} = \frac{\partial}{\partial \mu_u} - \frac{\partial}{\partial \mu_d} = \frac{\partial}{\partial \mu_u} + \frac{\partial}{\partial \mu_{\bar{d}}} \quad (10)$$

For degenerate systems of  $u$  and  $d$  quarks,

$$\frac{\partial C_{u\bar{d}}}{\partial \mu_S} = \frac{\partial C_{u\bar{d}}}{\partial \mu_u} - \frac{\partial C_{u\bar{d}}}{\partial \mu_{\bar{d}}} = 0 \quad (11)$$

at  $\mu_u = \mu_d = 0$ . In this study we analyze  $\partial / \partial \mu_V$  which gives non-trivial results even with degenerate quarks. In the following  $\partial / \partial \mu$  stands for  $\partial / \partial \mu_V$ .

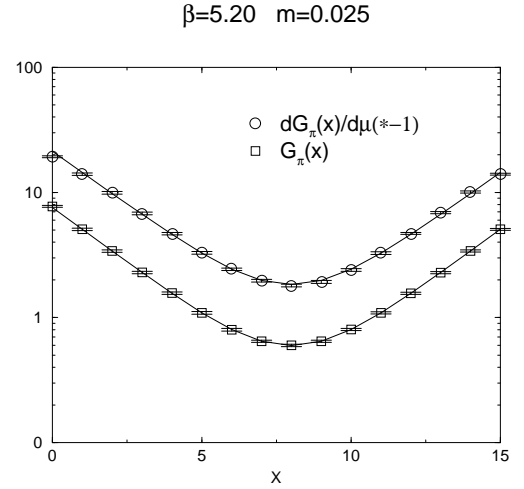


Figure 1. The pion correlation function,  $G_\pi(x)$  and its derivative with respect to the chemical potential,  $\frac{\partial G_\pi(x)}{\partial \mu}$  at  $\beta = 5.20$ .  $\frac{\partial G_\pi(x)}{\partial \mu}$  gives negative values. To plot them in logarithmic scale, they are multiplied by -1. Single pole fitting results are also shown, represented by solid lines.

### 4. Preliminary results

We present preliminary results of  $\partial m / \partial \mu$  for  $N_f = 2$  staggered quarks. Simulations are done on a lattice of size  $16 \times 8 \times 8 \times 4$  at  $m_q = 0.025$  with  $\beta = 5.20, 5.26, 5.32$  and  $5.34$ . We use the R-algorithm to generate configurations. The finite temperature transition occurs at  $\beta \approx 5.28$

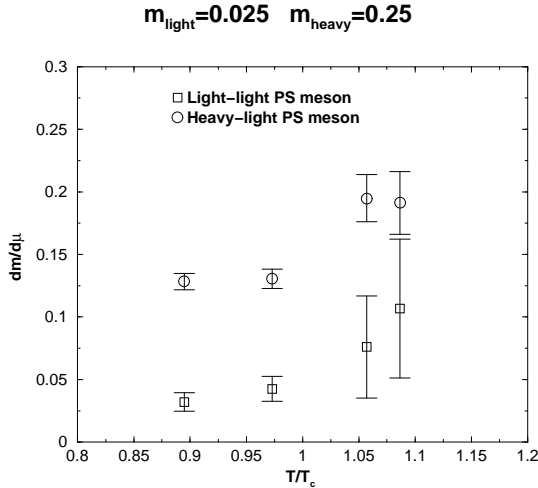


Figure 2.  $\partial m / \partial \mu$  of light-light and heavy-light pseudoscalar mesons as a function of  $T/T_c$ .

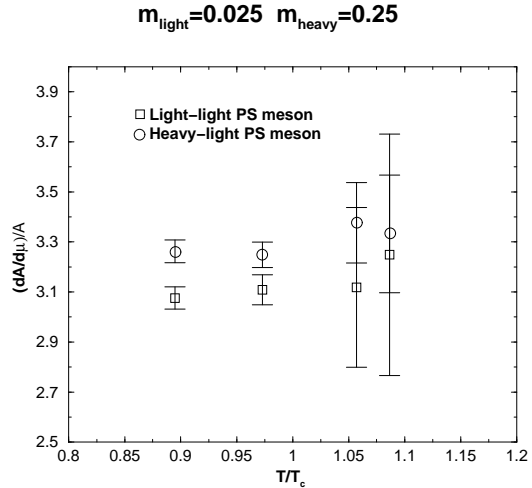


Figure 3. Response of the coupling  $A$  to chemical potential,  $\partial \ln A / \partial \mu$  as a function of  $T/T_c$ .

[3] and the above  $\beta$  values are translated to  $T/T_c = 0.90, 0.97, 1.06$  and  $1.09$  respectively.

We measure the pion screening mass. The quark propagator is calculated with  $m_q = 0.025$  (light) and  $0.25$  (heavy). Then we construct the pion correlator with light-light and light-heavy quarks.

Fig. 1 shows the pion (light-light) correlation function  $G_\pi(x)$  and its derivative with respect to  $\mu$  at  $\beta = 5.20$ . We perform single pole fit for the data, which turned out to be sufficient for the pion channel.

Fig. 2 shows  $\partial m / \partial \mu$  as a function of  $T/T_c$ . Despite the large errors we observe a systematic tendency towards raising the derivative of  $m$  above  $T_c$ .

Fig. 3 shows the response of the coupling  $A$ ,  $\partial \ln A / \partial \mu$  as a function of  $T/T_c$ . Both light-light and heavy-light mesons show similar values and no appreciable temperature dependence.

## 5. Discussions

Our preliminary results show remarkable characteristics of the response of meson masses to chemical potential. Possible interpretations for  $\partial m / \partial \mu$  of the light-light system are as follows.

The weak response of the mass below  $T_c$  indicates a persistence of the Nambu-Goldstone boson nature at least up to  $T = 0.97T_c$ . Growth of it above  $T_c$  is consistent with chiral restoration since the meson loses the Nambu-Goldstone character.

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## REFERENCES

1. T.Hatsuda and S-H.Lee, Phys. Rev. C46 (1992) 34.
2. S. Gottlieb, W. Liu, R. L. Renken, R. L. Sugar, and D. Toussaint, Phys. Rev. D38 (1988) 2888.
3. S. Gottlieb, W. Liu, R. L. Renken, R. L. Sugar, and D. Toussaint, Phys. Rev. D35 (1988) 3972.